STAT 2593

Lecture 027 - Intervals Based on Normal Populations

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Intervals Based on Normal Populations

Learning Objectives

1. Construct confidence intervals for the mean in small n normal populations with σ^2 unknown.


ANOTHER ONE

Small Sample Size with Unknown Variance

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  - This will have a t **distribution** with $n-1$ degrees of freedom, denoted t_{n-1}.

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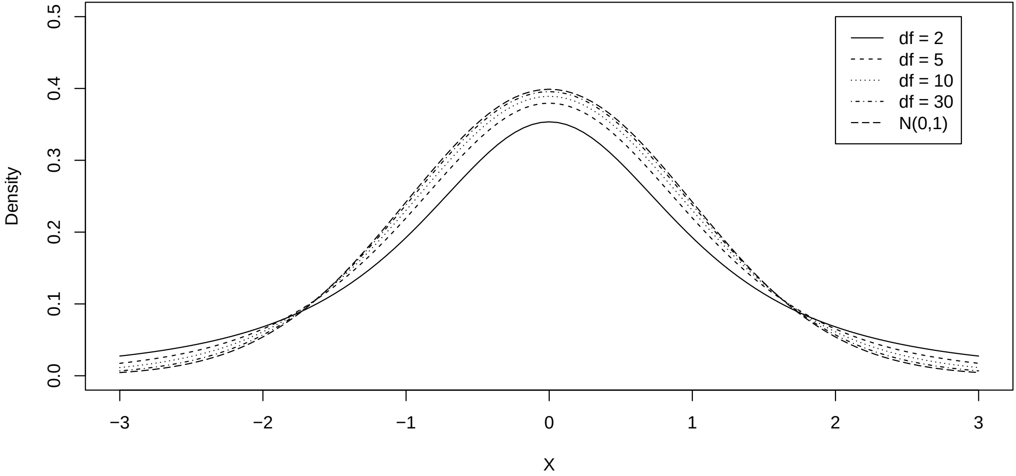
- As the number of degrees of freedom, ν, grows, the normal distribution is approached.

The t Distirbution, Visually



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- The resulting interval estimator will remain $\overline{X} \pm t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$.
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- This is only true *if* the population is assumed to be normal.

Summary

- When variance is unknown, but the population is still assumed to be normal, and we have small sample sizes, we need to use a t distribution to form the confidence intervals.
- The t distribution will work for large samples as well, but the normal approximation will not work in small samples.
- The t distribution approaches the normal distribution as $\nu \rightarrow \infty$ (where $\nu = n - 1$ in this case).